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A NON-LINEAR LAW OF THE DEFORMATION OF SOFT SOILS†

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Using a model of an elastic and viscoelastic (standard-linear) solid, a non-linear equation of state is constructed to model the volume deformation of soft soils. The moduli of bulk compression are assumed to be non-linear functions of a parameter which characterizes the structural failure of the soil. It is shown that this law describes observed effects of the bulk compression of soil. The results of a parametric analysis of the equation of state are given. © 1998 Elsevier Science Ltd. All rights reserved.

The most complete of the existing models of the deformation of soil-type media is that given in [1, 2], which takes into account all the basic mechanical properties which affect the dynamic processes (non-linear and irreversible volume deformability, elastoplastic shear and the dependence of the yield stress on pressure during shear) as well as the rheological properties. A detailed review of research on the equation of state and experimental justification for the soil model can be found in [3].

A model of the bulk deformation of soil which takes into account the viscous properties of the medium has been described in [4, 5]. The equations of state proposed in [9], which take the relaxation properties of the medium into account, are based on the results of experimental investigations on the dynamic deformation of soil [6–9] and a development of the model proposed in [1, 2].‡

The model proposed in [1, 2] was further improved in [4–7], mainly by refining and perfecting the law of bulk deformation; the law of shear deformation was left unchanged. Models which take account of more complex experimental factors have also been proposed [6–8].

The non-linear law of the bulk deformation of soil with variable moduli of bulk compression proposed below is based on the approach previously followed in developing the laws of interaction between solids and soil [10, 11].

1. PRINCIPLES OF THE CONSTRUCTION OF THE PROPOSED EQUATION OF STATE

Since soil cannot withstand any substantial tensile stresses, only compressive stresses will be considered. It has been shown by experiment [4–9] that when soil in its natural state is compressed, its structural bonds are destroyed. When the compressive stress reaches a certain value, the structural bond fails completely. This gives rise to dilatancy effects (depending on the relations between the main compressive stresses), and there is a change in the density and therefore the volume of the soil, as well as the values of its mechanical parameters. The original values of the bulk modulus of the soil change especially during deformation. It has been found by experiment that the value of the shear modulus can change by an order of magnitude or more during shear deformation of soil [10, 11].

In this paper we consider the law of bulk deformation, assuming that the bulk modulus of the soil is a function of the structural failure of the soil.

It is assumed that the structural bonds of the soil start to fail from the very beginning of compression, and fail completely when the bulk deformation θ reaches a value θ_* (the values of the bulk deformation θ and pressure P are assumed to be positive during compression). The degree of failure (fractionation) is assumed to depend on the rate at which the load is applied to the soil. In some cases, the fractured soil starts to become more dense as the pressure increases during failure, and the bulk modulus of the soil K increases up to a value of the bulk deformation $\theta = \theta_{**}$, but any further increase in compression has no effect on the mechanical parameters of the soil.

†*Prikl. Mat. Mekh.* Vol. 62, No. 3, pp. 503–511, 1998.

‡ See also KULINICH, Yu. V., NAROZHNAYA, Z. V. and RYKOV, G. V., The mechanical characteristics of sandy and clay soils taking into account their viscoplastic properties under short-term dynamic loads. Preprint No. 69. Inst. Problem Mekh., Akad. Nauk SSSR, Moscow, 1976.

It should be noted that when the bulk deformation has reached the value $\theta = \theta_{**}$, the only soils for which the shear modulus increases are those in which, after failure, the structural bonds are renewed between the particles during compression. The increase in the bulk modulus also depends on the moisture content of the soil. For instance, after the total failure of loess soils with 15–20% moisture content, an increase in pressure results in compaction of the soil and accordingly an increase in its density, which in turn leads to an increase in the bulk moduli of the soil. This also occurs in loams, sandy loams and sands with 15–20% moisture content.

An increase in the compression of dry soils and rock after failure does not always lead to an increase in the bulk modulus of the medium. Soil and rocks with a low moisture content (granite, basalt, marble, tuff, etc.) do not bind or stick together after total failure, but remain in their fractured state. In such cases, the bulk modulus does not increase as the compression increases.

We will therefore consider two versions of the law of bulk deformation: for dry and brittle soils, which do not stick together after failure, and for compacted, moist soils.

On the assumption that the bulk modulus is a function of the failure of the medium, the law of bulk compression of the soil is non-linear.

In its general form, the law of deformation of the soil is [1, 2]

$$P = f(I_s, \theta), \quad \frac{d\tilde{S}_{ij}}{dt} + \lambda S_{ij} = 2Ge_{ij}$$

$$\lambda = \frac{1}{2J_2}\left(2GW - \frac{dJ_2}{dt}\right), \quad W = S_{ij}e_{ij}, \quad J_2 = \frac{1}{2}S_{ij}S_{ij} \tag{1.1}$$

Here S_{ij}, e_{ij} are deviators of the stress tensor and deformation rates respectively, the tilde denotes the Jaumann derivative [2], the functional $\lambda > 0$ for $J_2 = (J(P))^2$, $\lambda \equiv 0$ for $J_2 < (J(P))^2$, $J(P)$ is a function which defines the generalized condition of Mises flow [6, 7] and G is the shear modulus, which is related to the bulk modulus by the well-known equation

$$G = K(1 - 2\mu)/(1 + \mu) \tag{1.2}$$

where μ is Poisson's ratio of the soil.

We take $I_s \in [0, 1]$ as the parameter characterizing the extent of fracture of the soil. The bulk modulus is assumed to be a function of the fracture parameter $K = K(I_s)$. Hence, by (1.2), the shear modulus of the soil is also a function of the fracture parameter.

Certain conditions, to be considered below, are imposed on the equation $P = f(I_s, \theta)$ in (1.1).

We will now consider the laws of bulk compression of a soil medium in the light of the above assumptions. Shear deformations of the soil are described by the same equations (1.1) proposed in [2].

2. A NON-LINEAR ELASTOPLASTIC LAW OF BULK DEFORMATION OF SOIL

The model of a linearly elastic medium is used as the basic equation of state. If the bulk modulus is a function of structural failure of the soil, the equation of state of the medium has the form

$$P = K_e(I_s)\theta \ \text{ for } d\theta/dt \geqslant 0 \tag{2.1}$$

$$P = K_R(I_s)\theta \ \text{ for } d\theta/dt < 0 \tag{2.2}$$

For brittle, dry soils, the quantity $K_e(I_s)$ is given by the relation

$$K_e(I_s) = K_* \exp(\alpha(1 - I_s)), \quad I_s = \theta/\theta_* \tag{2.3}$$

For soft, compacting, moist soils with $0 \leqslant \theta \leqslant \theta_*$ and $dP/dt < 0$

$$K_e(I_s) = K_t \exp(\beta(I_s - 1)), \quad I_s = \theta/\theta_t \tag{2.4}$$

For soil of these types with $0 \leqslant \theta \leqslant \theta_*$ and $dP/dt \geqslant 0$, relation (2.3) holds.

The unloading modulus varies as follows:

$$K_R(I_s) = K_{RN} \exp(\gamma(I_s - 1)), \quad I_s = \theta / \theta_R \tag{2.5}$$

In relations (2.1)–(2.5) P is the compression, θ is the actual value of the deformation, θ_* is the deformation at which there is complete structural failure of the soil, θ_t is the deformation for $dP/dt < 0$, $0 < \theta_t < \theta_*$, θ_R is the deformation corresponding to the start of unloading for $d\theta/dt < 0$, α, β, γ are dimensionless parameters characterizing the degree of change of the moduli of compression and unloading $K_e K_R$, respectively, K_* is the value of the bulk modulus of completely fractured soil, K_t is the value of the bulk modulus of soil for $\theta = \theta_t$, K_{RN} is the initial value of the unloading modulus.

We can make a qualitative analysis of the equation of state (2.1) and (2.2) by constructing graphs of the compression of the soil $P(\theta)$. To do so, we determine the pressure P, specifying the change of deformation in the form

$$\theta = \theta_m \sin(\pi t/T) \tag{2.6}$$

where θ_m is the maximum deformation, t is the time and T is the half-period of the change of deformation,

We will first consider the $P(\theta)$ curves for brittle soils. We use Eqs (2.1)–(2.3) in this case. The graph of the compression of soil for model parameter values $K_* = 10$ MPa; $\alpha = 2.5$; $\beta = 0.1$; $\gamma = 1.0$; $K_{RN} = K_* \exp\alpha$ and $\theta_m = 0.175$; $T = 0.4$ is shown in Fig. 1.

Curves 1–3 in Fig. 1 refer to the values $\theta_* = 0.3$; 0.25 and 0.2, respectively. For all values of θ_*, the pressure increases with deformation, reaches a maximum, and then decreases. After the deformation has reached the maximum value $\theta = \theta_m$, unloading begins. According to Eq. (2.2), unloading is also non-linear. At every stage, the graphs of the bulk and unloading moduli $P(\theta)$ are decreasing functions of the deformation in the given case. Equations (2.1) can thus describe the "descending" part of the graph observed experimentally. As Fig. 1 shows, the parameter θ_* characterizes the structural strength of the soil. A decrease in θ_* leads to a decrease in the maximum pressure.

By varying the values of the model parameters α, β, γ, K_*, K_{RN} one can obviously obtain a set of graphs $P(\theta)$ with different values of the maximum pressure and residual deformation. Specific parameters values must be determined experimentally for specific types of soil. The methods used to find α, β and γ are described in [10]. The values of K_* and K_{RN} are found as described, for example, in [4, 5].

Essentially, the bulk moduli found from Eqs (2.3)–(2.5) are the secant moduli of the graph $P(\theta)$.

In fact, while the soil is undergoing structural failure during compression, its mechanical properties change. Thus, each value of the deformation θ corresponds to a definite state of the soil material and, therefore, a specific value of the bulk modulus corresponds specifically to a given state. If the given state were the final state of the soil, the value of the modulus would then remain constant. However, it is obvious that no such state exists. The structural state of soil can always be altered by applying different loads.

The values of the mechanical parameters of the fractured structure are usually determined experimentally. One must start from this position when setting up the equations of the model. However, Eqs (2.3)–(2.5) can be expressed in different forms.

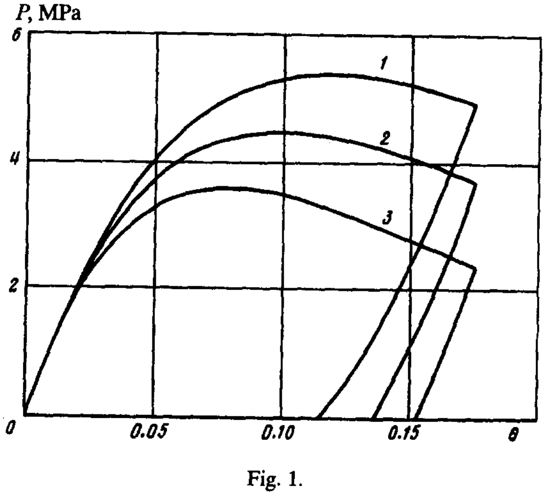


Fig. 1.

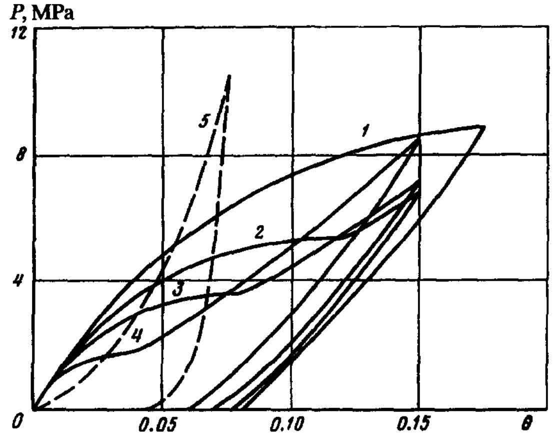


Fig. 2.

Figure 2 shows the curves $P(\theta)$ in the case where relations (2.3)–(2.5) are satisfied. Curves 1–5 correspond to the values $\theta_* = 0.5; 0.3; 0.2; 0.1; 0.01$. For curve 5, $K_{RN} = 10K_* \exp \alpha$. The values of the other parameters of the model are unchanged.

The compression graphs are qualitatively different in this case: there is no "descending" section, and after a certain increment of deformation, the pressure and unloading start to increase rapidly when $d\theta/dt < 0$.

These graphs correspond to soft compacting soils, such as loess with a 15–25% moisture content [4–9]. In such cases the soil structures tarts to break down under the effect of compression, but failure stops after a certain time. The structural changes occur as the bulk modulus increases. In fact, the soil starts to compact and strengthens. The bulk modulus of the soil begins to increase from a time $dP/dt < 0$. corresponding to the transition point on the graph of $P(\theta)$, where $\theta = \theta_t$. As we can see from Fig. 2, the value of θ depends on θ_*. As θ_* decreases, the value of Θ_t also decreases (curves 1–4). When $\theta_* = 0.01$, which denotes insignificant structural strength of the soil, the $P(\theta)$ curve corresponds to the graph of fractured soil (curve 5).

Obviously here too, we can obtain variations of the $P(\theta)$ curve corresponding to the results of specific experiments by varying the values of the model parameters.

As $\theta_* \to \infty$ the equations of state of the soil (2.1) and (2.2) become the model of a linearly elastic medium.

The model considered does not take into account temporal effects, the viscous properties of the soil, and so on. We will therefore allow for these parameters using the model of a linear viscoelastic body.

3. A NON-LINEAR VISCOPLASTIC LAW OF DEFORMATION OF SOIL

We will consider a linear viscoelastic (standard linear) solid, and assume that the bulk moduli are also functions of the structural changes in the soil. Then the equations of state of the soil have the form

$$K_D^{-1}(I_s)\frac{dP}{dt} + K_s^{-1}(I_s)\mu_0(I_s)P = \frac{d\theta}{dt} + \mu_0(I_s)\theta \quad \text{for} \quad d\theta/dt \geqslant 0 \tag{3.1}$$

$$K_R^{-1}(I_s)\frac{dP}{dt} = \frac{d\theta}{dt} \quad \text{for} \quad d\theta/dt < 0 \tag{3.2}$$

where $K_D(I_s)$, $K_s(I_s)$, $K_R(I_s)$ are functions of the dynamic and static compression and unloading, respectively, which characterize the changes of these moduli with the fracture parameter I_s, and μ_0 is a parameter of the volume viscosity of the soil.

The functions of the change in the dynamic and static bulk moduli of the soil $K_D(I_s)$ and $K_s(I_s)$ for brittle soils in the range $0 \leqslant \theta \leqslant \theta_*$ are defined by the relations

$$K_D(I_s) = K_{D*}\exp(\beta(1 - I_s)),\ \ K_s(I_s) = K_{s*}\exp(\alpha(1 - I_s)) \tag{3.3}$$

where K_{D*} and K_{s*} are, respectively, the moduli of dynamic and static compression of structurally-fractured soil, β and α are dimensionless indices characterizing the degree of change of the bulk moduli of the soil, $I_s = \theta/\theta_*$ and θ_* is the value of the deformation at which the soil experiences total structural failure.

According to (3.3), the initial values of the bulk moduli are

$$K_{DN} = K_{D*}\exp\beta,\ \ K_{sN} = K_{s*}\exp\alpha \tag{3.4}$$

Hence we have

$$\alpha = \beta + \ln(\gamma_* / \gamma_N),\ \ \gamma_* = K_{D*} / K_{s*},\ \ \gamma_N = K_{DN} / K_{SN} \tag{3.5}$$

In most cases the values of K_{s*} are determined experimentally, and the value of K_{D*} remains unknown. Thus, in order to determine the ratio of the moduli of dynamic and static compression for structurally-fractured soil, we use the expression

$$\gamma_* = \gamma_N + (\gamma_m - \gamma_N)(\mu_N^{-1} d\theta / dt)^{\varkappa},\ \ \mu_N = K_{DN}K_{sN} / [(K_{DN} - K_{sN})\eta] \tag{3.6}$$

where γ_m is the maximum possible value of this parameter for the given form of soil, μ_N is the viscosity parameter for the soil with its initial structure, η is the viscosity coefficient and $\varkappa$ is a dimensionless exponent.

According to (3.6), γ_* depends on the rate of deformation. In fact, breakdown of the soil structure and its fractionation are assumed to depend on the rate of loading of the medium. The greater the extent of breakdown of the soil structure (fractionation), the larger the value of γ_* will be.

The change of the viscosity parameter with change in soil structure is defined by the relation

$$\mu_0(I_s) = \mu_*\exp(\alpha^0(1 - I_s)),\ \ \mu_* = \mu_N / \gamma_*,\ \ \alpha^0 = \ln(\gamma_*) \tag{3.7}$$

where α^0 is a dimensionless index characterizing the degree of change of the viscosity parameter as a function of the breakdown of the soil structure.

The values of the parameters β, γ_m, $\varkappa$ are found by experiment. The values of the other parameters of the model can then be determined using (3.3)–(3.7).

In this version of the model, according to Eq. (3.1), the soil continues to break down up to the instant when $\theta = \theta_*$, when $d\theta/dt \geqslant 0$. This is true of brittle soils.

The graphs of $P(\theta)$ obtained in experiments do not have a "descending" section for compacted moist soils. In such cases the functions of the model (3.1) are defined as follows:

$$K_D(I_s) = K_{Dt}\exp(\beta_t(I_s - 1)),\ \ K_s(I_s) = K_D(I_s)/\gamma_{st}\ \text{ for }\ 0 \leqslant \theta \leqslant \theta_*,\ \ dP/dt < 0 \tag{3.8}$$

Here $I_s = \theta/\theta_t$; θ_t, K_{Dt}, D_{st} are the values of the corresponding parameters for $dP/dt < 0$. For $0 \leqslant \theta \leqslant \theta_*$ and $dP/dt \geqslant 0$, relations (3.3) are satisfied.

Relations (3.8) imply that once the condition $dP/dt < 0$ is satisfied, the pressure starts to increase and continue to increase as long as the deformation is increasing. The pressure can only increase up to a certain value of the deformation θ_{**}. An increase of pressure and deformation will suppress breakdown of the soil structure, that is, will increase the secant moduli of the soil, and γ_{st} decreases. The actual value of γ_{st} is given by the expression

$$\gamma_{st} = \gamma_{st0} - (\gamma_{st0} - \gamma_{**})\left(\frac{\theta - \theta_t}{\theta_{**}}\right)^{\beta_0} \tag{3.9}$$

where γ_{**} is the limiting value of γ_{st} when $\theta = \theta_{**}$, γ_{st} is the value of γ_{st} for $dP/dt < 0$ and β_0 is a dimensionless exponent.

Once the deformation θ_{**} has been attained, the parameters of the model remain constant. The values of θ_{**}, γ_{**} and β_0 are determined experimentally.

As the non-linear elastoplastic model, the unloading modulus $K_R(I_s)$ is defined by the relation

$$K_R(I_s) = K_{RN}\exp(\beta_R(I_s - 1)),\ \ I_s = \theta / \theta_R \tag{3.10}$$

where θ_g is the value of the deformation at time $d\theta/dt < 0$, K_{RN} is the initial value of the unloading modulus and β_R is a dimensionless index characterizing the degree of change of the unloading modulus.

We will consider the behaviour of the soil described by Eqs (3.1) and (3.2), using relation (2.6) to describe the change in deformation.

Figure 3 shows how the pressure changes with the deformation for brittle soils. The graphs for $P(\theta)$ were obtained with model parameter values: $\beta = 0.5$; $K_{s*} = 10$ MPa $\gamma_N = 1.1$; $\gamma_m = 10$; $\theta_* = 0.05$; $\beta_R = 3.0$ and loads $\theta_m = 0.15$; $T = 0.4$ s. Curves 1–5 correspond to values $\mu_N = 10, 20, 50, 100$ and $1000\ \text{s}^{-1}$.

We can see from Fig. 3 that an increase in the viscosity parameter μ_N, which corresponds to a reduction in the viscosity coefficient, will lead to a substantial quantitative change in the curves $P(\theta)$. An increase in the viscosity coefficient leads to an increase in the loss of mechanical energy, determined correspondingly by the area of the graph $P(\theta)$. Hence this should lead to an increase in the absorption coefficient of the soil, as found in experiments. An increase in the viscosity coefficient will also lead to a clear manifestation of the "descending" part of the graph $P(\theta)$.

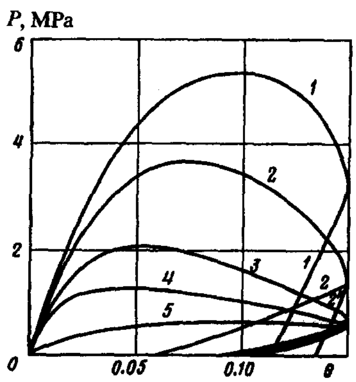
The change in pressure for different values of μ_N shows that the model is quite sensitive to changes in the viscosity parameter (Fig. 3).

Different unloading curves can be obtained by varying the values of β_R and K_{RN}. The unloading branch 1 and 2′ is obtained for $\beta_R = 0.5$ and $K_{RN} = 5K_{DN}$.

Fixing the values $\mu_N = 1000\ \text{s}^{-1}$, we will consider the influence of a change of deformation rate on the curves of $P(\theta)$ (Fig. 4).

Curves 1–7 in Fig. 4 correspond to the values $T = 4 \times 10^{-6}, 4 \times 10^{-5}, 4 \times 10^{-4}, 4 \times 10^{-3}, 4 \times 10^{-2}$ and 10 s. Clearly, a change in T which, according to Eq. (2.6), corresponds to a change in the deformation rate, has a substantial effect on the graph of $P(\theta)$. Curves 1–7 in Fig. 4 roughly correspond to the values of the deformation rate $\theta = d\theta/dt = 10^6, 10^3, 10^4, 10^3, 10^2$, 4 and $0.4\ \text{s}^{-1}$.

According to the given equation of state of the soil for values of $\theta = 1$ and $10\ \text{s}^{-1}$ the graphs of $P(\theta)$ are almost identical (curves 6, 7 Fig. 4). As the deformation rate increases further, there is a substantial change in the shape of the curves $P(\theta)$. The absorption properties of the soil also intensify.



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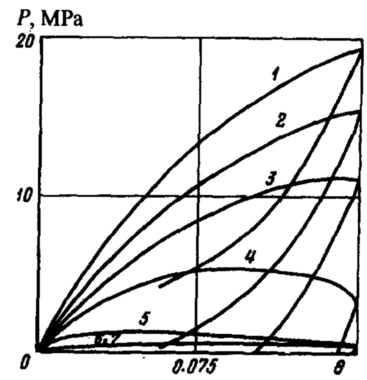

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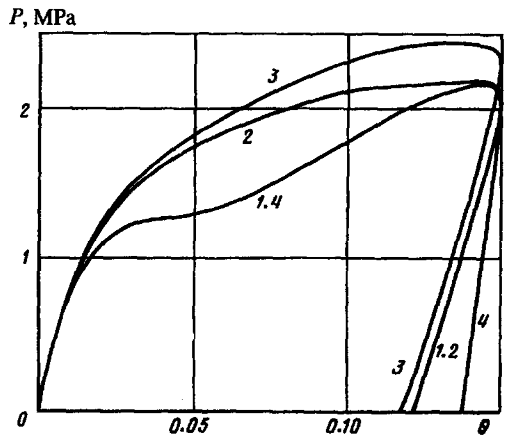

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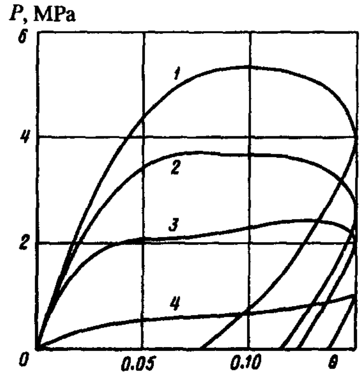

Fig. 6.

According to the results of calculations, a change in the values of the parameters β, γ_N, γ_m, K_{s*}, θ_* also affects the shape of the curves. The change in the curves $P(\theta)$ for different values of the parameters of the model are on the whole consistent with experimental results [4–9].

The shape of the curves of $P(\theta)$ is qualitatively different for soft compacting soils from those for brittle soils.

Figure 5 shows curves of $P(\theta)$ for soft soils taking into account relations (3.8) and (3.9) for the following parameter values: $\beta_t = 0.1$; $\beta_0 = 0.7$; $\gamma_{**} = 2$ and $\mu_N = 100\ s^{-1}$. The values of the other parameters were unchanged.

Curves 1–3 (Fig. 5) correspond to the values $\theta_* = 0.05$; 0.1 and 0.5. Curve 4 was obtained with $K_{RN} = 5K_{DN}$.

The $P(\theta)$ curves for soft soils show a change in sign of the curvature, as discussed in [1, 2]. An increase in θ_* will lead to an increase in the absorption properties of the soil, since the process of soil breakdown is protracted and requires a larger expenditure of mechanical energy.

We will consider the influence of changes of the viscosity parameter or viscosity (coefficient) on the shape of the $P(\theta)$ curves for fixed values of the parameters of the model β, γ_N, γ_m, K_{s*}, β_t, β_0, γ_{**}, θ_* and $\theta_m = 0.15$, $T = 0.4$ s.

Figure 6 shows the curves obtained for values of the viscosity parameter $\mu_N = 10, 20, 50, 100\ s^{-1}$ (curves 1–4). An increase in μ_N (a decrease in the viscosity coefficient η) leads to a reduction in the absorptivity of the soil. No changes in the sign of curvature in the $P(\theta)$ curves are observed for large values of the viscosity coefficients (curve *1*). When the viscosity parameters increase, the structure breaks down at the initial stages of compression and this is accompanied by an increase in the bulk moduli of the soil as described by relations (3.8).

We analysed the effect of a change of deformation rate on the behaviour of the $P(\theta)$ curves at values of $\theta = d\theta/dt = 10^6, 10^4, 10^2, 1\ s^{-1}$ (curves 1–4, Fig. 7). The value of the viscosity parameter for these curves was fixed: $\mu_N = 1000\ s^{-1}$. An increase in the deformation rate also increases the absorption properties of the soil. A rise in the deformation rate is also accompanied by fracture of the soil at early stages of compression so that no changes are observed in the sign of the curvature of $P(\theta)$.

By also varying the values of the parameters of Eq. (3.10), one can obtain different branches of unloading, and therefore different values of the residual deformations.

The results of the calculations show that the proposed non-linear laws of deformation of the soil are, on the whole, in qualitative agreement with the results of experimental investigations of soil tests [4–9]. Note that there is no need to introduce an additional equation into the model to describe the "descending" part of $P(\theta)$ curve, as in [4]. The deformation of the soil before the unloading stage is described completely by the single equation (3.1). As we can see from Figs 3–7, the equations of state (3.1) and (3.2) take full account of the basic properties of the soils.

In the same way, equations of state which allow for breakdown of the soil structure can be formulated for shear deformation of the soil medium.

However, since bulk deformation prevails over shear deformation, we have confined ourselves here to a more detailed consideration of the laws of bulk deformation of the soil.

4. DILATANCY

The dilatancy properties of soil develop when the structure breaks down [8]. Allowance for dilatancy should be made as follows.

On the usual assumption of soil mechanics, the bulk deformation is taken to consist of two parts

$$\theta = \theta_P + \theta_S \tag{4.1}$$

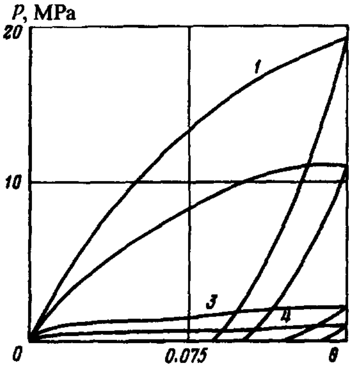

Fig. 7.

where θ is the total bulk deformation, θ_P is the bulk deformation corresponding to the action of the spherical part of the stress tensor and θ_S is that corresponding to the action of a deviator of the stress tensor.

The value of θ_P is found from the above laws of bulk deformation of the soil. The component arising from shear deformations is determined from the relation [8]

$$\theta_S = \lambda_D \varepsilon \tag{4.2}$$

where λ_D is the dilatancy coefficient and ε is the shear deformation.

The dilatancy coefficient can be defined by the expression

$$\lambda_D = (1 - I_s)(\dot{\varepsilon} / \mu_s)^{\omega}, \quad I_s = \varepsilon / \varepsilon_*, \quad \dot{\varepsilon} = d\varepsilon / dt \tag{4.3}$$

where ε_* is the limiting value of the shear deformation at which there is total structural failure of the soil during pure shear, μ_s is the parameter of shear viscosity of the soil and ω is a dimensionless exponent.

The values of ε_*, μ_s and ω are determined experimentally.

It is clear from (4.2) and (4.3) that the soil volume increases with shear deformation ε. This continues up to the deformation value $\varepsilon = \varepsilon_*$, after which, according to (4.2) and (4.3), the soil becomes denser and therefore decreases in volume. The diminution in volume of the soil can be restricted by any limiting value of the shear deformation $\varepsilon = \varepsilon_{**}$, beyond which it can be assumed that the shear deformation has no effect on the change in volume of the soil.

In cases where the spherical component exceeds the deviator part of the stress tensor during compression of the soil, the dilatancy properties of soils can be neglected [6, 7].

The laws of the bulk deformation of soils proposed above are therefore an improvement on, and supplement, the equation of state of soils (1.1) proposed in [1, 2], giving a better description of their behaviour, while allowing for the basic properties, including rheological effects, observed experimentally.

The graphs of the bulk compression of soils obtained by means of a parametric analysis of the laws of deformation are quite consistent with experimental results [4–9], thus confirming that the laws can be used to solve applied problems of soil mechanics. The proposed non-linear laws may also apply to the behaviour of materials whose structure changes during deformation.

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